

## Experimental Method for Complex Thermo-mechanical Material Analysis

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**Abstract** An experimental system for complex thermo-mechanical material analysis is presented in this article. The system provides simultaneous measurements of tensile properties as well as heat generation in the process of tensile deformation. The cooling curve of the sample after its reversible deformation was measured. On the basis of an exponential model of a cooling body with respect to the Biot number  $Bi$ , it is possible to calculate the specific heat capacity  $c_p$ , the thermal diffusivity  $\alpha$ , and the thermal conductivity  $k$ . The method had been tested on a variety of materials and the results were compared to those in the technical literature or obtained by reference independent experiments and showed very good agreement.

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## 1 Introduction

Many physical processes such as heating, cooling, and radioactive decay have an exponential trend, which is described by an exponential equation. This means that, although the temperature of a body will get closer to that of its surroundings, the two temperatures will never be equal. The experimental determination of the Biot number and thermal diffusivity in highly conductive materials subjected to laminar air flow were presented in [1]. The tensile test is formally standardized by ISO 527-1-2 ([http://www.ides.com/property\\_descriptions/ISO527-1-2.asp](http://www.ides.com/property_descriptions/ISO527-1-2.asp)). On the other hand, the creation of heat in the process of material deformation contains information about thermal properties of the material under investigation, which is relatively hidden. It is necessary to find a proper complementary method applied to a tensile test, which reveals these parameters. Thermography with the application of a proper mathematical algorithm could be the desired solution. The fundamentals and methods of infrared thermography are widely described in [2].

In this article, we deal with the presentation of mechanical and thermal material parameters from a simultaneous tensile test and thermovision inspection of the same investigated sample according to the theoretical model of a cooling body—lumped capacitance method. The values of the specific heat capacity  $c_p$ , the thermal diffusivity  $\alpha$ , and the thermal conductivity  $k$  together with the mechanical values, tensile strength  $\sigma_{UTS}$ , yield stress  $\sigma_y$ , proportional limit  $\sigma_E$ , ductility  $A$ , and Young's modulus of elasticity  $E$  were measured.

## 2 Theory

Phenomena in fluid flow and heat transfer depend on dimensionless parameters. These parameters give information concerning the relevant flow regimes of a given solution. Casting equations in dimensionless form helps to show the generality of application to a broad class of situations (rather than just one set of dimensional parameters). It is a generally good practice to use dimensionless numbers, equations, and results for presentation whenever possible.

In our case, we apply the Biot number. For  $Bi \gg 1$ , the convection heat transfer process offers little resistance to heat transfer. For  $Bi \ll 1$ , the conduction heat transfer process offers little resistance to heat transfer.

The application of the model of a cooling body depends on the value of the Biot number expressed in the form [3,4],

$$Bi = \frac{hV}{Sk}, \quad (1)$$

where  $k$  is the thermal conductivity,  $V$  is the sample volume,  $h$  is the total coefficient of heat transfer, and  $S$  is the surface area accessible to the heat flow.

The temperature law is described by

$$T = T_{\infty} + (T_0 - T_{\infty}) e^{-\frac{t}{\tau}}, \quad (2)$$

where  $T_{\infty}$  is the ambient temperature,  $T_0$  is the initial temperature of the cooling process, and  $\tau$  is the relaxation time [4] (<http://www.math.okstate.edu/~noell/labs/cooling.pdf>) that can be expressed through the characteristic thermal parameters of the investigated sample in the form,

$$\tau = \frac{\rho c_p V}{hS}, \quad (3)$$

where  $\rho$  is the density of the sample and  $c_p$  is the specific heat at constant pressure. The relationship among all three thermal parameters is given by [4]

$$\alpha = \frac{k}{\rho c_p} \quad (4)$$

where  $\alpha$  is the thermal diffusivity ([http://en.wikipedia.org/wiki/Thermal\\_diffusivity](http://en.wikipedia.org/wiki/Thermal_diffusivity); <http://en.wikipedia.org/wiki/Thermalconductivity>).

The procedure for determination of the thermal constants is as follows. The measured dependence of the cooling curve obtained after deformation of the sample is interpolated in the Matlab environment by Eq. 2. It is necessary to set limiting values for  $T_0$ ,  $T_{\infty}$ , and  $\tau$  (estimated from the cooling curve). The “fit quality” was tested by using statistical functions offered by Matlab (see below). After finding proper values of  $T_0$ ,  $T_{\infty}$ , and  $\tau$ , we insert them into Eq. 2 and we express  $\tau$  according to Eq. 3 (in this way, we obtain a function of parameters  $h$  and  $c_p$ ). Subsequently, we repeat the interpolation procedure in order to obtain  $h$  and  $c_p$ . After obtaining values of  $h$  and  $c_p$  (estimated from an appropriate interval of values or from independent measurements), we introduce to Eq. 2 the obtained values of  $h$  and  $c_p$  by using Eq. 4 and finally we obtain a function for determination of parameters  $\alpha$  and  $k$ . A fit of all parameters is completed after obtaining the best values of the following statistical parameters.

A first statistical parameter *SSE* (summed square of residuals) measures the total difference of the values from the interpolation procedure and measured values. Values of this statistical parameter are always positive and should be close to zero.

A second statistical parameter *R-square* ( $R^2$ ) indicates how successful the fit is in explaining the variation of the data. It is also called the square of the multiple correlation coefficient and the coefficient of multiple determination. The value of this statistical parameter should be close to one.

A third statistical parameter “*Adjusted R-Square*” ( $Adj R^2$ ) uses the *R-square* statistics and adjusts it on the basis of the residual degrees of freedom. The residual degrees of freedom are defined as the number of response values  $n$  minus the number of fitted coefficients  $m$  estimated from the response values. The adjusted *R-square* parameter is generally the best indicator of the fit quality when you add additional coefficients to the model. The adjusted *R-square* parameter can take on any value less than or equal to one, with a value closer to one indicating a better fit.

Finally, a statistical parameter root-mean-squared error (*RMSE*) is also known as the fit standard error and the standard error of the regression. An *RMSE* value close to zero indicates a better fit.

### 3 Experimental Technique

The experimental apparatus consists of a thermovision camera VIGO THERM V-20 system with a sensitivity in the investigated temperature range equal to  $0.01\text{ }^{\circ}\text{C}$ . The mechanical deformation of the sample was realized by the tensile test machine, Hounsfield H20K-W. The deformation of the sample was recorded by extensometer PS25C from Tinius Olsen.

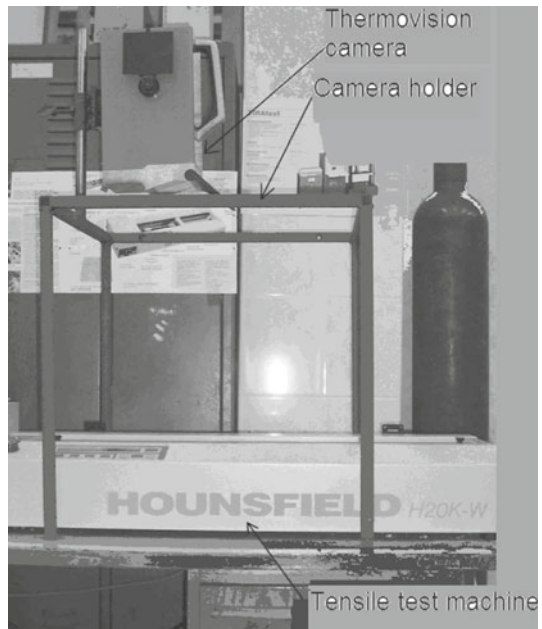
The thermovision camera was in the perpendicular direction to the deformed (pulled) sample (see Fig. 1).

The experiment was realized in two steps. The first step was a standard tensile test, which determines the values of the tensile strength  $\sigma_{\text{UTS}}$ , yield stress  $\sigma_y$ , proportionality limit  $\sigma_E$ , ductility  $A$ , and Young's modulus of elasticity  $E$ .

Then the sample was pulled and maintained at a standard length. The sample temperature rises because of deformation. From the cooling curve monitored by the thermovision camera, the specific heat capacity  $c_p$ , the thermal diffusivity  $\alpha$ , and the thermal conductivity  $k$  are obtained.

To test our method for highly conductive samples, in the first step, we would like to present the results of mechanical and thermophysical properties for samples of pure copper ( $k = 401\text{ W}\cdot\text{m}^{-1}\cdot\text{K}^{-1}$ ) (<http://en.wikipedia.org/wiki/Thermalconductivity>),

**Fig. 1** Experimental setup (tested sample is fixed in horizontal direction by Hounsfield tensile test machine, perpendicular to thermovision camera view)



and subsequently an aluminum alloy, which is representative of a moderately conductive material, with the composition 99.6% Al, 0.3% Fe, 0.05% Pb, 0.03% Ti;  $k = 120 \text{ W} \cdot \text{m}^{-1} \cdot \text{K}^{-1}$  to  $180 \text{ W} \cdot \text{m}^{-1} \cdot \text{K}^{-1}$  (<http://en.wikipedia.org/wiki/Thermalconductivity>). Thermal properties of steels were studied in detail by neural networks in [5].

As a representative low conductivity material, we used a sample of a rubber blend ( $k \sim 0.1 \text{ W} \cdot \text{m}^{-1} \cdot \text{K}^{-1}$ ). The density of samples was measured by application of the Archimedes principle in isopropanol alcohol.

Metallic samples for the tensile test were prepared according to STN 10002-1. Rubber samples for the tensile test were prepared according to STN ISO 37.

We obtained the reference values of the thermal properties of rubber blend using the flash method described in [6].

#### 4 Experimental Results and Discussion

The first tested sample was copper. Table 1 shows the mechanical properties of copper. The yield stress was evaluated as an offset yield stress  $R_{p,0.2}$  ([http://en.wikipedia.org/wiki/Tensile\\_strength](http://en.wikipedia.org/wiki/Tensile_strength)). Young's modulus was measured four times for three different samples of copper. The average Young's modulus for all copper samples is  $E = (105.98 \pm 8.92) \text{ GPa}$ , which is in good agreement with literature value. We can see that the obtained results are in good agreement with experimental values [7].

We then measured the thermophysical properties of a copper sample. We calculated the Biot number; its value is equal to  $4.292 \times 10^{-6}$  and we can see that its value meets the targeted limit introduced above. In Table 2, we have shown literature and experimental values of thermal parameters for copper. Estimated uncertainties of mechanical and thermal properties determination are less than 10%.

The agreement of both sets of values is very good, which supports the validity of the used model. This fact is also supported by the set of statistical parameters of the model presented in Table 3.

The next tested sample was an aluminum alloy. In this case, we present only thermal parameters. From the value of the Biot number, it is clearly seen that the exponential

**Table 1** Mechanical properties of copper: literature and experimental values [7]

Copper	$\sigma_{UTS}$ (MPa)	$\sigma_y$ (MPa)	$A$ (%)	$E$ (GPa)
Literature values	55–330	230–380	10–50	110–120
Experimental values	213.8	258.2	–	105.5

**Table 2** Thermophysical properties of copper [7]

Copper	$\alpha \times 10^4$ ( $\text{m}^2 \cdot \text{s}^{-1}$ )	$c_p$ ( $\text{J} \cdot \text{kg}^{-1} \cdot \text{K}^{-1}$ )	$k$ ( $\text{W} \cdot \text{m}^{-1} \cdot \text{K}^{-1}$ )
Literature values	1.165	385	410
Experimental values	1.192	385	410.3
$\text{Bi} = 4.292 \times 10^{-6}$			

**Table 3** Statistical parameters of model

<i>SSE</i>	$R^2$	<i>AdjR</i> <sup>2</sup>	<i>RMSE</i>
0.0028	0.9174	0.9174	0.0142

**Table 4** Experimental and literature thermophysical properties for an aluminum alloy—99.6% Al, 0.3% Fe, 0.05% Pb, 0.03% Ti [7]

Physical parameter	Experimental	Literature
$\alpha \times 10^5$ ( $\text{m}^2 \cdot \text{s}^{-1}$ )	9.97	9.92
$c_p$ ( $\text{J} \cdot \text{kg}^{-1} \cdot \text{K}^{-1}$ )	880	896
$k$ ( $\text{W} \cdot \text{m}^{-1} \cdot \text{K}^{-1}$ )	237	240
$Bi = 2.55 \times 10^{-5}$		

**Table 5** Mechanical parameters of the tested rubber blend obtained from tensile test

Material	$E$ (MPa)	$\sigma_E$ (MPa)	$\sigma_y$ (MPa)	$\sigma_{UTS}$ (MPa)	$A$ (%)
Rubber blend	4.03	0.4	2	8.82	525

**Table 6** Thermophysical properties for a rubber sample using both flash and CTMA methods

Thermal parameter	CTMA/flash method
$\alpha \times 10^7$ ( $\text{m}^2 \cdot \text{s}^{-1}$ )	1.475/1.465
$c_p$ ( $\text{J} \cdot \text{kg}^{-1} \cdot \text{K}^{-1}$ )	1507.7/1500
$k$ ( $\text{W} \cdot \text{m}^{-1} \cdot \text{K}^{-1}$ )	0.265/0.262
$Bi = 0.04$	

**Table 7** Statistical parameters of the fit of experimental data with an exponential model

<i>SSE</i>	$R^2$	<i>AdjR</i> <sup>2</sup>	<i>RMSE</i>
0.0023	0.9325	0.9325	0.0112

model described above could also be valid in this case. Results of measurements of the thermophysical parameters of the aluminum alloy are reported in Table 4 together with literature values. From the obtained results, it is seen that there is very good agreement of both literature and experimental values.

To test the model for low conductivity materials, we present results for the mechanical and thermal parameters of a rubber blend. Mechanical parameters of the blend are reported in Table 5, and they correspond to characteristic values for rubber blends.

The thermal parameters of the rubber blend were tested independently by the flash method described in [6]. The results are collected in Table 6.

The statistical parameters of the model are shown in Table 7.

## 5 Conclusions

From the presented results, it is possible to draw the following conclusions. complex thermo-mechanical material (CTMA) is based on the exponential model of a cooling

body. The validity of the model was tested experimentally. However, in every case, we also calculated the value of the Biot number as a crucial criterion of the model validity. The obtained results are in very good agreement with those obtained from independent measurements or from comparison with literature values.

The presented method is quick and reliable for complex measurements and evaluation of mechanical and thermal parameters of solid materials. All thermal experimental data were statistically tested and the defined statistical parameters gave sound evidence of the reliability of the model. In a comparison of the present method with others, the relative technical simplicity and very good precision are observed. Simultaneous measurements of mechanical and thermal properties are an advantage of this technique.

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